SOLUTIONS

PROBLEM SET 1

CHM 3411, Dr. Chatfield

1. (a) $$\lim_{\lambda\to\infty} \rho = \lim_{\lambda\to\infty} \frac{8\pi hc}{\lambda^5}\left(e^{hc/\lambda kT}-1\right)^{-1}$$

$$= \lim_{\lambda\to\infty} \frac{8\pi hc}{\lambda^5}\Big(1+\frac{hc}{\lambda kT}+\underset{\substack{\uparrow \\ \text{higher-order terms,} \\ \text{unimportant for } \lambda\to\infty}}{\ldots}-1\Big)^{-1} = \frac{8\pi kT}{\lambda^4}$$

(b) To find λ_{max}, set $\frac{d\rho}{d\lambda}=0$ and solve for λ

$$0 = \frac{d\rho}{d\lambda} = \frac{d}{d\lambda}\,\frac{8\pi kT}{\lambda^5}\left(e^{hc/\lambda kT}-1\right)^{-1}$$

$$= \frac{-40\pi hc}{\lambda^6}\left(e^{hc/\lambda kT}-1\right)^{-1} + \frac{8\pi hc}{\lambda^5}\left(\frac{hc}{\lambda^2 kT}\right)e^{hc/\lambda kT}\left(e^{hc/\lambda kT}-1\right)^{-2}$$

$$\Big\Downarrow \times \frac{\lambda^7}{8\pi hc}\left(e^{hc/\lambda kT}-1\right)^2$$

$$= -5\lambda\left(e^{hc/\lambda kT}-1\right) + \frac{hc}{kT}\,e^{hc/\lambda kT}$$

$$\lambda_{max} = \frac{hc}{5kT}\,\frac{e^{hc/\lambda_{max}kT}}{e^{hc/\lambda_{max}kT}-1} \longrightarrow \boxed{\frac{hc}{5kT}} \text{ for } e^{hc/\lambda_{max}kT} >> 1$$

Plug in to check $\Rightarrow e^{hc/\lambda_{max}kT} = e^5 >> $ ✓

(c) This follows immediately:

$$T\lambda_{max} = \frac{\overset{6.626\times10^{-34}\,\mathrm{Js}\quad 3\times10^{8}\,\mathrm{m\,s^{-1}}}{hc}}{\underset{\substack{\uparrow \\ 1.38\times10^{-23}\,\mathrm{J\,K^{-1}}}}{5k}} = \boxed{2.88\times10^{-3}\,\mathrm{mK} = A_w}$$

$\updownarrow$ good agreement

Look up online: $A_w = 2.898\times10^{-3}\,\mathrm{mK}$

2. $$E_{k,max} = h\overset{\frac{c}{\lambda}\,\swarrow}{\nu} - \overset{h\nu_0\,\swarrow}{\Phi}$$

$$\Phi = \frac{hc}{\lambda} - E_{k,max} = \frac{(6.626\times10^{-34}\,J\,s)(2.99\times10^{8}\,m\,s^{-1})}{254\times10^{-9}\,m} - 0.42\,eV$$

$$= 7.80\times10^{-19}\,J - 0.42\,eV = \boxed{7.13\times10^{-19}\,J = 4.45\,eV}$$

$\nwarrow \nearrow$

convert using $1\,eV = 1.60218\times10^{-19}\,J$

$$\nu_0 = \frac{\Phi}{h} = 1.07561\times10^{15}\,s^{-1}$$

$$\boxed{\lambda_0 = \frac{c}{\nu_0} = 278\,nm}$$

3. $\overset{\text{Exercise}}{[7A.9(a)]}$ $\lambda = \frac{h}{p} = \frac{h}{mv}$

$$\boxed{v = \frac{h}{m\lambda} = 0.024\,m\,s^{-1}}$$

$m = m_e = 9.109\times10^{-31}\,kg$, $h = 6.626\times10^{-34}\,J\,s$,

$\lambda = 3.0\times10^{-2}\,m$

4. Below is a longer explanation that I expected, but it gives you an idea of how to craft intuitive explanations for non-experts.

Around 1900, most scientists thought that science had developed explanations for essentially all physical phenomena, and the universe was basically understood – at least the laws by which it worked. But then, as technology advanced and more and more precise measurements were made, discrepancies between scientific theory and actual measurements started cropping up. Eventually, they simply could not be ignored, and scientists were driven to develop new theories that could explain all of the evidence.

The experiments may seem obscure to a non-scientist, but they were compelling to scientists. The first is called blackbody radiation. All objects give off electromagnetic radiation (light) if they are heated, and the spectrum emitted can be measured. It turned out that when predictions were made based on the laws of nature as understood at the time, the predictions were absurd. They predicted that the energy radiated by a blackbody would be infinite (at least at short wavelength). A similar problem was encountered when heat capacities were measured (for certain solid substances). A heat capacity is the amount of energy it takes to raise the temperature of a substance by one degree. It different for different substances. It turns out that at very cold temperatures, especially as you approach absolute zero, the heat capacity gets very small and approaches zero. But the predictions based on the laws of nature at the time did not give that result. Something was wrong with the understanding of the laws of nature. There were other experiments that also gave scientists problems, but these give you a flavor.

Scientists were able to fix the predictions by making bold, new assumptions and building new theories on them. One of those assumptions was that atoms and molecules can possess only particular values of energy, not just any value. These little packets of energy that separate one allowed energy level from another are called "quanta." It is as if the world were suddenly found out to be digital rather than analog: a shocking discovery! The second discovery is called the "wave-particle duality." Previously, small entities like electrons and protons had been assumed to behave essentially like billiard balls, while light had exclusively properties of waves (interference, refraction, and so on). But again, fine measurements showed that electrons displayed some properties of waves, and light displayed some properties of particles. Altogether, the discovery of energy quantization and the wave-particle duality compelled scientists to propose new laws governing nature at the very small scale. These laws are called quantum mechanics, and it has been demonstrated that they describe nature at the submicroscopic scale very well. To date there have been no discrepancies, in spite of the increasing sophistication of experimental measurements.