PART 1 Common integrals

Algebraic functions

A.1 $$\int x^n dx = \frac{x^{n+1}}{n+1} + \text{constant}, \quad n \neq -1$$

A.2 $$\int \frac{1}{x} dx = \ln x + \text{constant}$$

Exponential functions

E.1 $$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}, \quad n! = n(n-1)\ldots 1;\ 0! \equiv 1$$

E.2 $$\int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2} dx = \frac{\pi^4}{15}$$

Gaussian functions

G.1 $$\int_0^\infty e^{-ax^2} dx = \frac{1}{2}\left(\frac{\pi}{a}\right)^{1/2}$$

G.2 $$\int_0^\infty x e^{-ax^2} dx = \frac{1}{2a}$$

G.3 $$\int_0^\infty x^2 e^{-ax^2} dx = \frac{1}{4}\left(\frac{\pi}{a^3}\right)^{1/2}$$

G.4 $$\int_0^\infty x^3 e^{-ax^2} dx = \frac{1}{2a^2}$$

G.5 $$\int_0^\infty x^4 e^{-ax^2} dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

G.6 $$\text{erf}\, z = \frac{2}{\pi^{1/2}} \int_0^z e^{-x^2} dx \qquad \text{erfc}\, z = 1 - \text{erf}\, z$$

G.7 $$\int_0^\infty x^{2m+1} e^{-ax^2} dx = \frac{m!}{2a^{m+1}}$$

G.8 $$\int_0^\infty x^{2m} e^{-ax^2} dx = \frac{(2m-1)!!}{2^{m+1} a^m}\left(\frac{\pi}{a}\right)^{1/2}$$

$$(2m-1)!! = 1 \times 3 \times 5 \cdots \times (2m-1)$$

Trigonometric functions

T.1 $$\int \sin ax\, dx = -\frac{1}{a} \cos ax + \text{constant}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + \text{constant}$$

T.2 $$\int \sin^2 ax\, dx = \frac{1}{2}x - \frac{\sin 2ax}{4a} + \text{constant}$$

T.3 $$\int \sin^3 ax\, dx = -\frac{(\sin^2 ax + 2)\cos ax}{3a} + \text{constant}$$

T.4 $$\int \sin^4 ax\, dx = \frac{3x}{8} - \frac{3}{8a}\sin ax \cos ax - \frac{1}{4a}\sin^3 ax \cos ax + \text{constant}$$

T.5 $$\int \sin ax \sin bx\, dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + \text{constant}, \quad a^2 \neq b^2$$

T.6 $$\int_0^L \sin nax \sin^2 ax\, dx = -\frac{1}{2a}\left\{\frac{1}{n} - \frac{1}{2(n+2)} - \frac{1}{2(n-2)}\right\} \times \{(-1)^n - 1\}$$

T.7 $$\int \sin ax \cos ax\, dx = \frac{1}{2a}\sin^2 ax + \text{constant}$$

T.8 $$\int \sin bx \cos ax\, dx = \frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + \text{constant}, \quad a^2 \neq b^2$$

T.9 $$\int x \sin ax \sin bx\, dx = -\frac{d}{da}\int \sin bx \cos ax\, dx$$

T.10 $$\int \cos^2 ax \sin ax\, dx = -\frac{1}{3a}\cos^3 ax + \text{constant}$$

T.11 $$\int x \sin^2 ax\, dx = \frac{x^2}{4} - \frac{x \sin 2ax}{4a} - \frac{\cos 2ax}{8a^2} + \text{constant}$$

T.12 $$\int x^2 \sin^2 ax\, dx = \frac{x^3}{6} - \left(\frac{x^2}{4a} - \frac{1}{8a^3}\right)\sin 2ax - \frac{x \cos 2ax}{4a^2} + \text{constant}$$

T.13 $$\int x \cos ax\, dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax + \text{constant}$$